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OXC-5807
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NRO REVIEW COMPLETED

October 3, 1963

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STAT Dear [redacted]

Enclosed please find three copies of the final report on the Resolution Improvement by Image Superposition under [redacted] STAT

Regards,

[redacted]

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Enclosures (3)

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COPY 2 OF 3

27 September 1963

RESOLUTION IMPROVEMENT BY IMAGE SUPERPOSITION

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The fact that photographic resolution is frequently limited by granularity, which is random, has led to the suggestion that photographs of a particular subject be superpositioned. The superpositioned set of photographs would thus have less grain noise, and, therefore, the set should have more information and a higher resolution. Cosmetic appearance and "information content" are also affected by superposition, but these are not presently subject to objective analysis and therefore are not discussed.

In this report, resolution improvement is shown to require image registration to better than one-quarter of the spatial period corresponding to the limiting resolution. Thus, to improve the resolution of a photograph with 50 cyc/mm (20 μ) limiting resolution, the second photograph would have to be aligned to the first with less than 5 μ misregistration.

In Figure 1, the solid curves are the exposure modulation (assumed gaussian) and the detectability modulation for a single photograph. The curves are normalized such that their intersection, which is the resolution limit for a high contrast target, occurs at the normalized spatial frequency $K/K_o = 1.0$.

The dashed curves are for two superpositioned transparencies. The detectability modulation is reduced by $(2)^{-\frac{1}{2}}$. The exposure modulation is reduced by 0.89 at $K = K_o$, which presumes the registration error, ϵ, to be $\epsilon = 0.15\ X_o$ where $X_o = 1/K_o$; and the ratio of the misregistered exposure modulation to the single transparency exposure modulation, $T_\epsilon\ (K/K_o)$, is

$$T_\epsilon\ (K) = \cos(\pi \epsilon K)$$

(See appendix for derivation.) This modulation transfer function is shown in Figure 2. As can be seen in Figure 1, the new intersection indicates a resolution gain of slightly less than 10%.

In Figure 3, the potential resolution gain is graphed as a function of registration error, assuming a gaussian transfer function. The gain is also a function of detectability modulation of the original transparency at $K = K_o$, and is drawn for several values. For the illustrated case, the larger potential gains can only be realized if the granularity is limiting performance such that the system's modulation transfer function is very high (e.g. < 0.5) at the limiting resolution.

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A different shape of system transfer function will alter the exact shape of the curves in Figure 3, but the general requirement for $\epsilon < 0.25\ X_0$ will still hold. As can be seen, only modest resolution gains are possible, and these only when superb registration can be achieved. 4, 8, 16, etc. photos can be handled by direct extension of this analysis.

Distortion and perspective errors are indistinguishable from registration errors, and these seriously affect aerial photography. It can be shown that the half angular field α over which registration is as good as ϵ is

$$\alpha = \tan^{-1}\left\{0.5\tan\varphi\left[1+\frac{\epsilon}{X_0}\theta\,\mathrm{ctn}\,\varphi\right] \pm 0.5\tan\varphi\left[\left(1+\frac{\epsilon}{X_0}\theta\,\mathrm{ctn}\,\varphi\right)^2+4\frac{\epsilon}{X_0}\theta\,\mathrm{ctn}^3\varphi\right]^{1/2}\right\}$$

where φ is the angular pointing or perspective difference in two photographs and θ is the camera's angular resolution.

Since φ must be small, $\tan\varphi = \varphi$. Also $\theta << \varphi$ is to be expected in any high acuity camera. Under these conditions,

$$\alpha = \tan^{-1}\frac{\varphi}{2}\left[1 \pm \left(1+\frac{4\epsilon\theta}{X_0\varphi^3}\right)^{1/2}\right]$$

and for α less than 10 deg.

$$\alpha_{TOT} \cong \alpha_{+} - \alpha_{-} = \varphi\left[1+\frac{4\epsilon\theta}{X_0\varphi^3}\right]^{1/2}.$$

Thus, for $\theta = 10^{-5}$ rad (2 arc-seconds resolution), $\varphi = 10^{-2}$ rad (½ deg pointing), and $\epsilon/X_0 = 0.1$ (registration-perspective error one-tenth of limiting detail size), the total angular field over which resolution gain is realized is only $\alpha_{TOT} = 1.3$ deg.

While image superposition may be useful to improve the resolution of low resolution photography or to improve the cosmetic and "information content" of any photography it is concluded that superposition is not attractive to improve resolution of high-acuity aerial photography.

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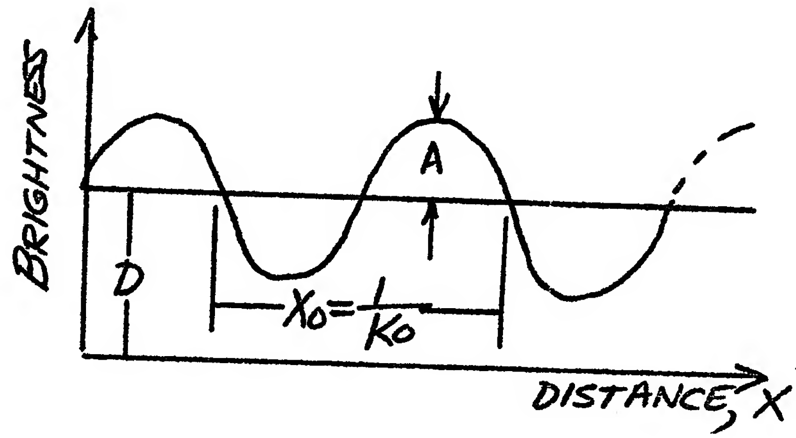
APPENDIX

Derivation of Misregistration Transfer Function



On the first transparency,

$$S_1 = D + A \sin\left(2\pi \frac{x}{x_0}\right).$$

If the second transparency is misregistered by ϵ,

$$S_2 = D + A \sin\left(2\pi \frac{x+\epsilon}{x_0}\right).$$

Thus

$$S_1 + S_2 = 2D + 2A \sin\left(2\pi \frac{x + \epsilon/2}{x_0}\right) \cos \frac{\pi\epsilon}{x_0}.$$

The modulation with $\epsilon = 0$ is

$$M_{1+2 \atop \epsilon=0} = \frac{2A}{2D},$$

and for $\epsilon \neq 0$ is

$$M_{1+2 \atop \epsilon\neq0} = \frac{2A}{2D} \cos \frac{\pi\epsilon}{x_0}.$$

Therefore the transfer function at $K = K_0$ is

$$T_\epsilon = \cos \frac{\pi\epsilon}{x_0}$$

and at other values is

$$T_\epsilon(k) = \cos \frac{\pi\epsilon}{x_0}\frac{k}{k_0} = \cos \pi\epsilon k$$

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FIG. 1

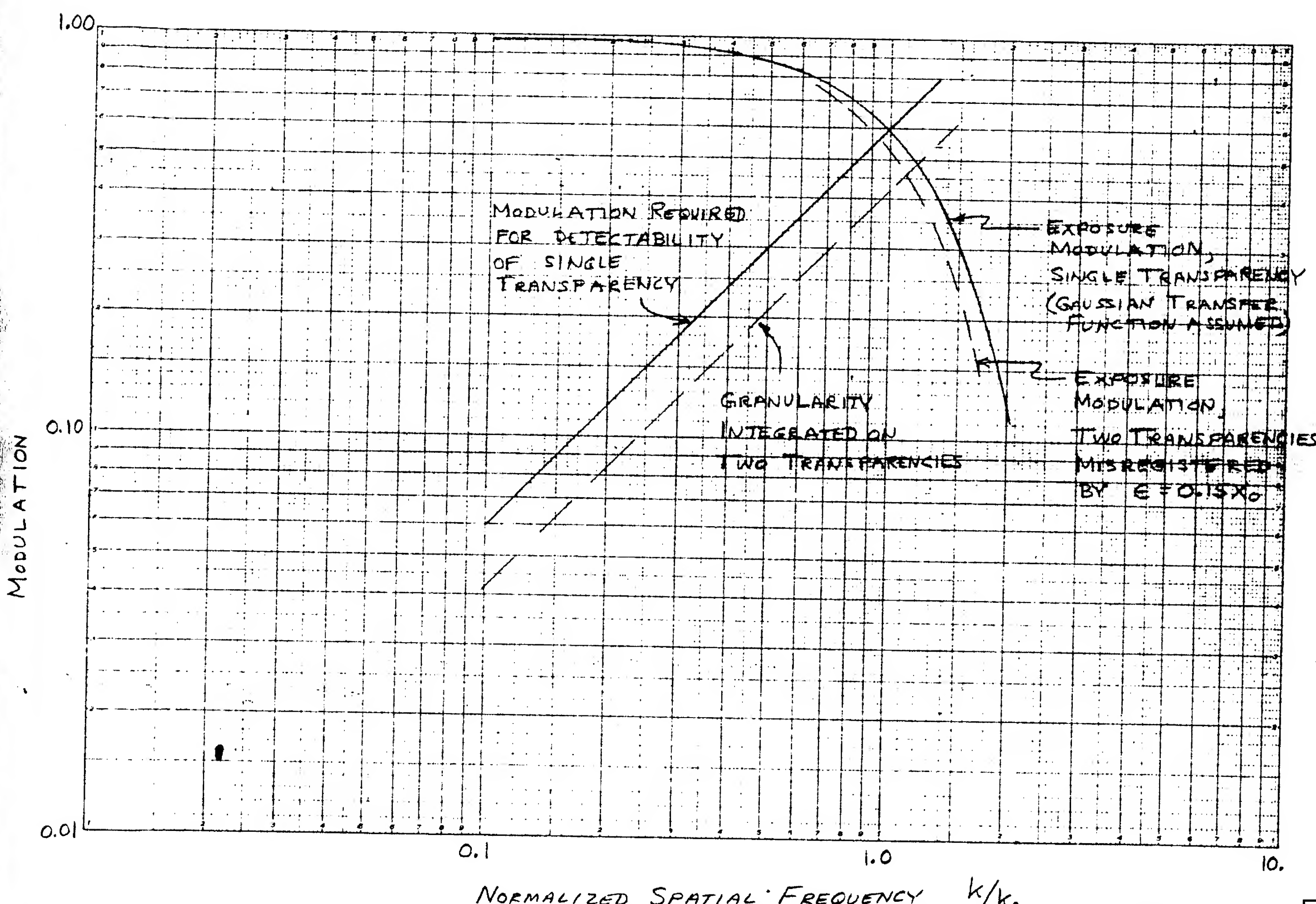

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FIG. 2

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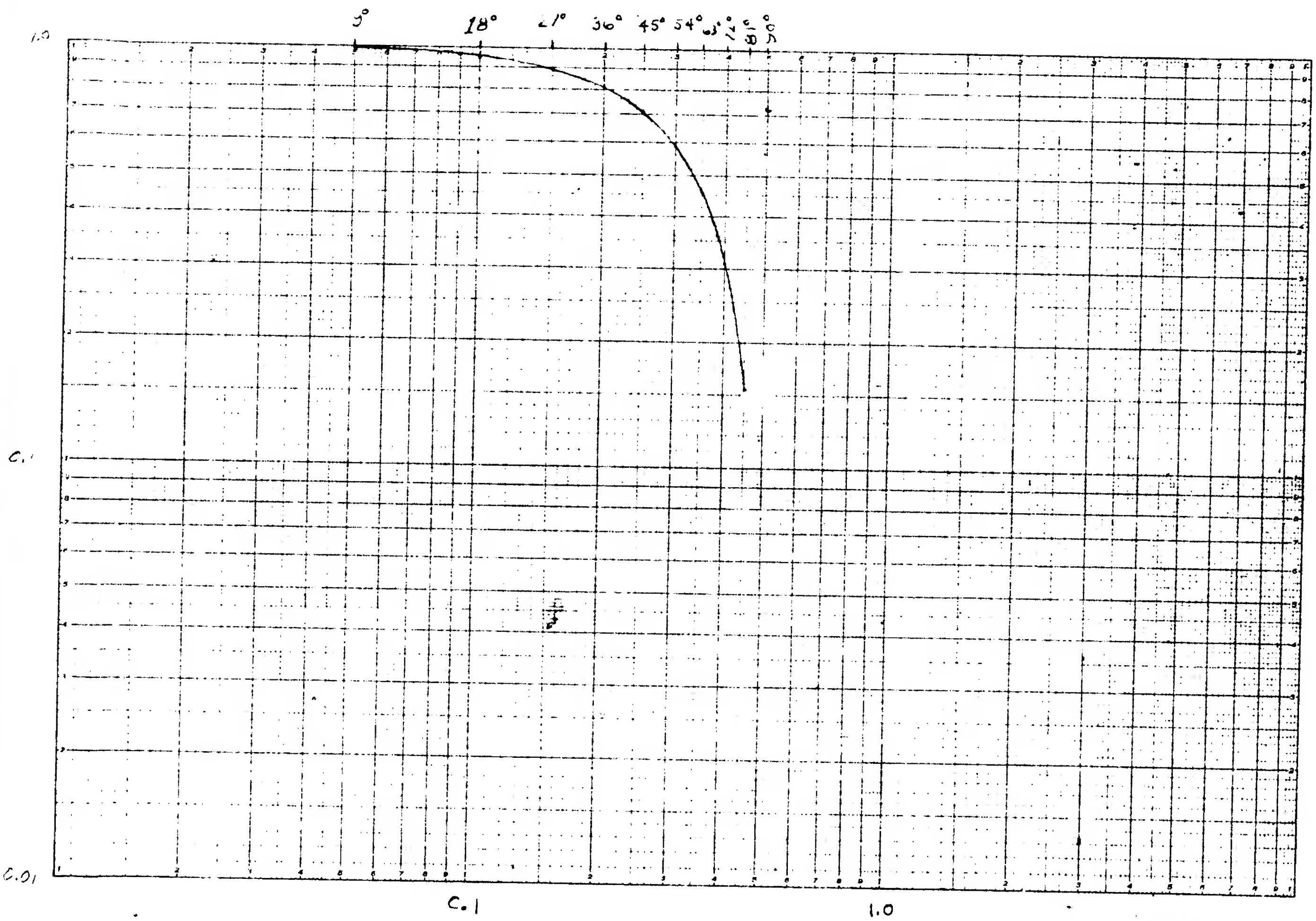
$T_\epsilon(k) = \cos\left[2\pi\left(\frac{\epsilon}{2x_0}\right)\frac{k}{k_0}\right]$



MISREGISTRATION $\left(\frac{\epsilon}{x_0}\right)$ (for $k = k_0$)

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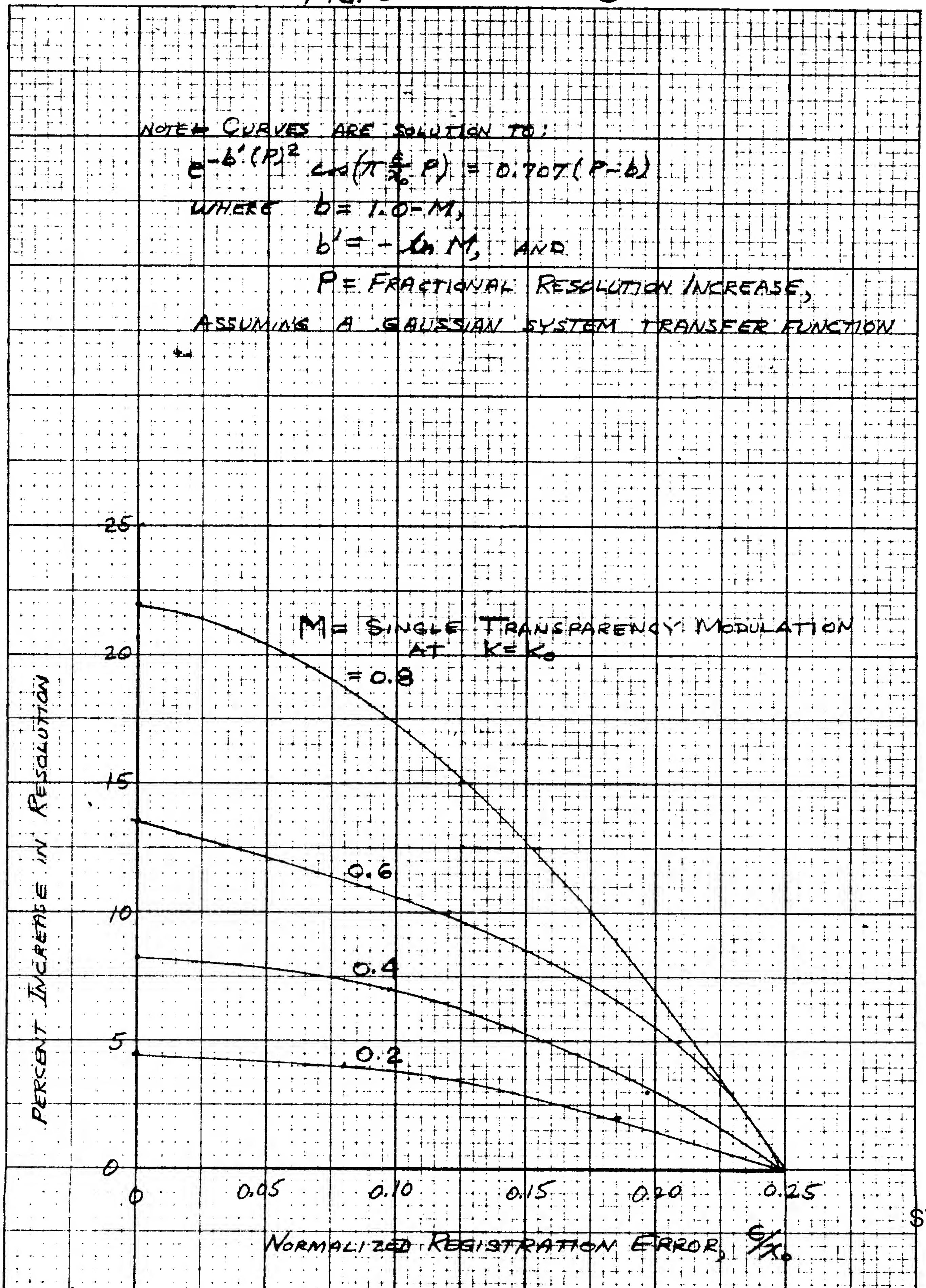
FIG. 3



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